CONCORDIA UNIVERSITY
DEPARTMENT OF COMPUTER SCIENCE & SOFTWARE ENGINEERING
COMP 232/2 Mathematics for Computer Science Fall 2015

Assignment 2 Due date: Monday October 12, 2015

1. For each of the arguments below, formalize them in propositional logic. If the argument is valid identify which inference rule was used, and formulate the tautology underlying the rule. If the argument is invalid, state whether the inverse or converse error was made.

   (a) All cheaters sit in the back row.
   George sits in the back row.
   $\therefore$ George is a cheater.

   (b) For all students x, if x studies discrete math, then x is good at logic.
   Dawn studies discrete math.
   $\therefore$ Dawn is good at logic.

   (c) If the compilation of a computer program produces error messages, then the program is not correct or the compiler is faulty.
   The compilation of this program does not produce error messages.
   $\therefore$ this program is correct and the compiler is not faulty.

   (d) All students who do not do their homework and do not study the course material will not get a good course grade.
   John gets a good course grade.
   $\therefore$ John did his homework or studied the course material.

2. For each of the premise-conclusion pairs below, give a valid step-by-step argument (proof) along with the name of the inference rule used in each step. For examples, see pages 73 and 74 in textbook.

   (a) Premise: $\{\neg p \vee q \to r,\ s \vee \neg q,\ \neg t,\ p \to t,\ \neg p \wedge r \to \neg s\}$, conclusion: $\neg q$.

   (b) Premise: $\{\neg p \to r \wedge \neg s,\ t \to s,\ u \to \neg p,\ \neg w,\ u \vee w\}$, conclusion: $\neg t \vee w$.

   (c) Premise: $\{p \vee q,\ q \to r,\ p \wedge s \to t,\ \neg r,\ \neg q \to u \wedge s\}$, conclusion: t.

3. Use rules of inference to show that

   (a)
   $$\begin{array}{l} \forall x\, \Big(R(x) \to \big(S(x) \vee Q(x)\big)\Big) \\ \exists x\, \big(\neg S(x)\big) \\ \hline \therefore \exists x\, \big(R(x) \to Q(x)\big) \end{array}$$

   (b)
   $$\begin{array}{l} \forall x\Big(P(x) \vee Q(x)\Big) \\ \forall x\Big(\big(\neg P(x) \wedge Q(x)\big) \to R(x)\Big) \\ \hline \therefore \forall x\Big(\neg R(x) \to P(x)\Big) \end{array}$$

4. Prove that the following four statements are equivalent:
   (a) n^2 is odd.
   (b) $1 - n$ is even.
   (c) n^3 is odd.
   (d) $n^2 + 1$ is even.

5. (a) Give a direct proof of: "If x is an odd integer and y is an even integer, then $x + y$ is odd."
   (b) Give a proof by contradiction of: "If n is an odd integer, then n^2 is odd."
   (c) Give a proof by contraposition of: "If n is an odd integer, then $n + 2$ is odd."

6. For each of the statements below state whether it is True or False. If True then give a proof. If False then explain why, e.g., by giving a counterexample.
   (a) For all positive $x, y \in \mathbb{R}$, if x is irrational and y is irrational then $x + y$ is irrational.
   (b) For all positive $x, y \in \mathbb{R}$, if x is irrational and y is rational then xy is irrational.
   (c) $\sqrt{3}$ is irrational.

7. Consider the statement concerning integers "*If $m + n$ is even, then $m - n$ is even.*"
   (a) Give a direct proof of the statement.
   (b) Give a proof by contraposition of the statement.
   (c) Prove the statement by contradiction.